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Tabella Integrali

Upgrade your brain

INTEGRALI

FONDAMENTALI

A cura di Gaetano Cioppa



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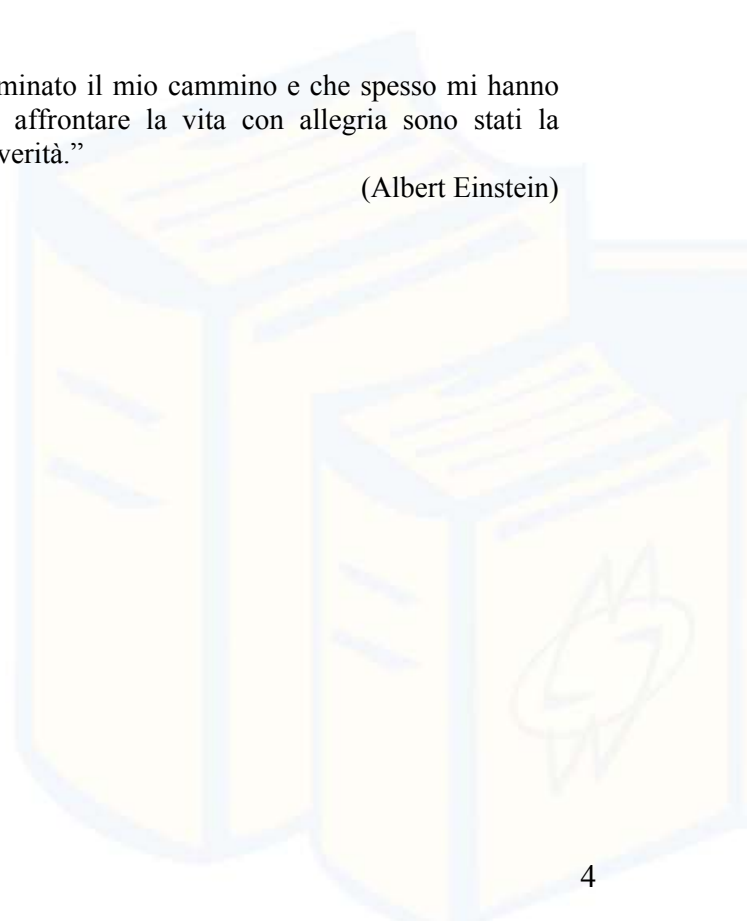


Upgrade your brain

Dedicato a Me

“Gli ideali che hanno illuminato il mio cammino e che spesso mi hanno dato nuovo coraggio per affrontare la vita con allegria sono stati la gentilezza la bellezza e la verità.”

(Albert Einstein)





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$$1. \int dx = x$$

$$2. \int x^\lambda dx = \frac{x^{\lambda+1}}{\lambda+1}$$

$$3. \int \sqrt[n]{x} dx = \frac{n}{n+1} \sqrt[n]{x^{n+1}}$$

$$4. \int \frac{1}{x^2+a^2} dx = \frac{1}{a} \operatorname{arctg} \left(\frac{x}{a} \right)$$

$$5. \int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right|$$

$$6. \int \frac{1}{a^2-x^2} dx = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right|$$

$$7. \int \frac{1}{\sqrt{x^2+a^2}} dx = \log \left| x + \sqrt{a^2+x^2} \right|$$

$$8. \int \frac{1}{\sqrt{a^2-x^2}} dx = \operatorname{arcsen} \left(\frac{x}{a} \right) = -\operatorname{arccos} \left(\frac{x}{a} \right)$$

$$9. \int \frac{1}{\sqrt{x^2-a^2}} dx = \log \left| x + \sqrt{x^2-a^2} \right|$$

$$10. \int \frac{1}{\sqrt{ax+b}} dx = \frac{2}{a} \sqrt{ax+b}$$

$$11. \int \frac{1}{ax+b} dx = \frac{1}{a} \log |ax+b|$$

$$12. \int \frac{1}{1+(ax+b)^2} dx = \frac{1}{a} \operatorname{arctg}(ax+b)$$

$$13. \int \frac{1}{x(ax+b)} dx = -\frac{1}{b} \log \left| \frac{ax+b}{x} \right|$$

$$14. \int \frac{1}{x\sqrt{x^2+a^2}} dx = -\frac{1}{a} \log \left| \frac{a+\sqrt{x^2+a^2}}{x} \right|$$

$$15. \int \frac{1}{x^2 \sqrt{x^2 + a^2}} dx = -\frac{\sqrt{x^2 + a^2}}{a^2 x}$$

$$16. \int \frac{1}{x(x^2 + a^2)} dx = \frac{1}{2a^2} \log \left(\frac{x^2}{x^2 + a^2} \right)$$

$$17. \int \frac{1}{x \sqrt{ax-1}} dx = 2 \operatorname{arctg} \sqrt{ax-1}$$

$$18. \int \frac{1}{(1+x^2)^2} dx = \frac{x}{2(1+x^2)} + \frac{1}{2} \operatorname{arctg} x$$

$$19. \int \frac{1}{x(x^2+1)} dx = \log \left| \frac{cx}{\sqrt{x^2+1}} \right|$$

$$20. \int \frac{1}{x^2(1+x)^2} dx = \log \left| \frac{x+1}{x} \right|^2 - \frac{2x+1}{x(1+x)^2}$$

$$21. \int \frac{1}{x^3+x^2} dx = -\frac{1}{x} + \log \frac{x+1}{x}$$

$$22. \int \frac{1}{x^6+x^4} dx = \operatorname{arctg} x + \frac{1}{x} - \frac{1}{3x^3}$$

$$23. \int \frac{1}{x^3-1} dx = \frac{1}{6} \log \left[\frac{(x-1)^2}{x^2+x+1} \right] - \frac{1}{\sqrt{3}} \operatorname{arctg} \left[\frac{2x+1}{\sqrt{3}} \right]$$

$$24. \int \frac{1}{x^3+1} dx = \frac{1}{6} \log \left[\frac{(x+1)^2}{x^2-x+1} \right] + \frac{1}{\sqrt{3}} \operatorname{arctg} \left(\frac{2x-1}{\sqrt{3}} \right)$$

$$25. \int \frac{1}{x^4-1} dx = \log \left(4 \sqrt{\left| \frac{x-1}{x+1} \right|} \right) - \frac{1}{2} \operatorname{arctg} x$$

$$26. \int \frac{1}{x^4+1} dx = \frac{\sqrt{2}}{4} \log \left(\sqrt{\frac{x^2+x\sqrt{2}+1}{x^2-x\sqrt{2}+1}} \right) + \frac{\sqrt{2}}{4} \left[\operatorname{arctg} \left(\frac{x\sqrt{2}}{1-x^2} \right) + k \right] + c$$

$$27. \int \frac{1}{x^2-x} dx = \log \left(\frac{x-1}{x} \right)$$

$$28. \int \frac{1}{x^2-1} dx = \frac{1}{2} \log \left| \frac{x-1}{x+1} \right| = -\operatorname{arccotgh} x$$

$$29. \int \frac{1}{\sqrt{x^2-1}} dx = \log \left(x + \sqrt{x^2-1} \right) = \text{settcosh } x$$

$$30. \int \frac{1}{1+x^2} dx = \text{arctg } x = -\text{arctg } x$$

$$31. \int \frac{1}{\sqrt{1+x^2}} dx = \log \left(x + \sqrt{x^2+1} \right) = \text{settsenh } x$$

$$32. \int \frac{1}{1-x^2} dx = \frac{1}{2} \log \left| \frac{x+1}{x-1} \right| = \text{setttgh } x = \text{arctgh } x$$

$$33. \int \frac{1}{\sqrt{1-x^2}} dx = \text{arcsen } x = -\text{arccos } x$$

$$34. \int \frac{x}{\sqrt{ax+b}} dx = \frac{2}{3a^2} (ax+b) \sqrt{ax+b}$$

$$35. \int \frac{x}{ax+b} dx = \frac{x}{a} - \frac{b}{a^2} \log |ax+b|$$

$$36. \int \frac{ax+b}{px+q} dx = \frac{a}{p}x + \frac{bp-aq}{p^2} \log |px+q| + c$$

$$37. \int \frac{x}{(ax+b)^2} dx = \frac{b}{a^2(ax+b)} + \frac{1}{a^2} \log |ax+b|$$

$$38. \int \frac{x}{x^2+a^2} dx = \frac{1}{2} \log (x^2+a^2)$$

$$39. \int \frac{x}{x^2-a^2} dx = \frac{1}{2} \log (x^2-a^2)$$

$$40. \int \frac{x}{a^2-x^2} dx = -\frac{1}{2} \log |a^2-x^2|$$

$$41. \int \frac{x}{\sqrt{x^2-a^2}} dx = \sqrt{x^2-a^2}$$

$$42. \int \frac{x}{\sqrt{x^2+a^2}} dx = \sqrt{x^2+a^2}$$

$$43. \int \frac{x}{\sqrt{a^2-x^2}} dx = -\sqrt{a^2-x^2}$$

$$44. \int \frac{x}{(a^2 - x^2)^n} dx = \frac{1}{2(n-1)(a^2 - x^2)^{n-1}}$$

$$45. \int \frac{x}{(a^2 + x^2)^n} dx = -\frac{1}{2(n-1)(a^2 + x^2)^{n-1}}$$

$$46. \int \frac{x}{(x^2 - a^2)^n} dx = -\frac{1}{2(n-1)(x^2 - a^2)^{n-1}}$$

$$47. \int \frac{x^2}{x^2 + a^2} dx = x - a \operatorname{arctg} \frac{x}{a}$$

$$48. \int \frac{x^2}{\sqrt{a^2 - x^2}} dx = \frac{a^2}{2} \operatorname{arcsen} \left(\frac{x}{a} \right) - \frac{1}{2} \sqrt{a^2 - x^2}$$

$$49. \int \frac{x^2}{\sqrt{x^2 + a^2}} dx = \frac{x}{2} \sqrt{x^2 + a^2} - \frac{a^2}{2} \log \left(x + \sqrt{x^2 + a^2} \right)$$

$$50. \int \frac{x^2}{\sqrt{x^2 - a^2}} dx = \frac{x}{2} \sqrt{x^2 - a^2} + \frac{a^2}{2} \log \left(x + \sqrt{x^2 - a^2} \right)$$

$$51. \int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log \left| x + \sqrt{x^2 + a^2} \right|$$

$$52. \int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log \left| x + \sqrt{x^2 - a^2} \right|$$

$$53. \int \sqrt{a^2 - x^2} dx = \frac{a^2}{2} \operatorname{arcsen} \frac{x}{a} + \frac{x}{2} \sqrt{a^2 - x^2}$$

$$54. \int x \sqrt{a^2 - x^2} dx = -\frac{\sqrt{(a^2 - x^2)^3}}{3}$$

$$55. \int x \sqrt{x^2 + a^2} dx = \frac{\sqrt{(x^2 + a^2)^3}}{3}$$

$$56. \int x \sqrt{x^2 - a^2} dx = \frac{\sqrt{(x^2 - a^2)^3}}{3}$$

$$57. \int x^2 \sqrt{a^2 - x^2} dx = \frac{x}{4} \left(x^2 - \frac{a^2}{2} \right) \sqrt{a^2 - x^2} + \frac{a^4}{16} \operatorname{arcsen} \frac{2x^2 - a^2}{a^2}$$

$$58. \int x^2 \sqrt{x^2 + a^2} dx = \frac{1}{4} (x^2 + a^2) x \sqrt{x^2 + a^2} - \frac{a^4}{16} \log \left[\left(x^2 + \frac{a^2}{2} \right) + x \sqrt{x^2 + a^2} \right]$$

$$59. \int x^2 \sqrt{x^2 - a^2} dx = \frac{1}{4} (x^2 - a^2) x \sqrt{x^2 - a^2} - \frac{a^4}{16} \log \left[\left(x^2 - \frac{a^2}{2} \right) + x \sqrt{x^2 - a^2} \right]$$

$$60. \int \frac{\sqrt{a^2 - x^2}}{x} dx = \sqrt{a^2 - x^2} - a \log \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right|$$

$$61. \int \frac{\sqrt{a^2 - x^2}}{x^2} dx = -\frac{1}{x} \sqrt{a^2 - x^2} - \arcsen \frac{x}{a}$$

$$62. \int x \sqrt{ax + b} dx = \frac{2(3ax - 2b)}{15a^2} \sqrt{(ax + b)^3}$$

$$63. \int \sqrt{ax + b} dx = \frac{2}{3a} \sqrt{(ax + b)^3}$$

$$64. \int \frac{\sqrt{1+x}}{x} dx = 2 \sqrt{x+1} + \log \frac{(\sqrt{x+1} - 1)^2}{x}$$

$$65. \int \frac{\sqrt{1+x}}{x^2} dx = \frac{1}{2} \log \left(\frac{\sqrt{1+x} - 1}{\sqrt{1+x} + 1} \right) - \frac{\sqrt{1+x}}{x}$$

$$66. \int \frac{\sqrt{x^2 - 1}}{x} dx = \sqrt{x^2 - 1} - \arctg \sqrt{x^2 - 1}$$

$$67. \int \frac{x}{\sqrt{1-x^4}} dx = \frac{1}{2} \arcsen x^2$$

$$68. \int \frac{x^2}{1-x^4} dx = \frac{1}{4} \log \left(\frac{x+1}{x-1} \right) - \frac{1}{2} \arctg x$$

$$69. \int \frac{x^3}{\sqrt{1-x^2}} dx = \frac{\sqrt{(1-x^2)^3}}{3} - \sqrt{1-x^2}$$

$$70. \int \sqrt{\frac{x}{1-x}} dx = \arctg \left(\sqrt{\frac{x}{1-x}} \right) - \sqrt{x-x^2}$$

$$71. \int \frac{x-1}{x+1} dx = x - 2 \log(x+1)$$

$$72. \int \sqrt{\frac{1-x}{1+x}} dx = \arcsen x + \sqrt{1-x^2}$$

$$73. \int x \sqrt{\frac{1-x}{1+x}} dx = \arctg \sqrt{\frac{1-x}{1+x}} + \frac{(x-2)}{2} \sqrt{1-x^2}$$

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$$74. \int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n+1)}$$

$$75. \int x(ax + b)^n dx = \frac{x(ax + b)^{n+1}}{a(n+1)} - \frac{(ax + b)^{n+2}}{a^2(n+1)(n+2)}$$

$$76. \int a^x dx = \frac{a^x}{\log a}$$

$$77. \int e^{ax} dx = \frac{e^{ax}}{a}$$

$$78. \int x a^x dx = \frac{x a^x}{\log a} - \frac{a^x}{\log^2 a}$$

$$79. \int x^2 a^x dx = \frac{x^2 a^x}{\log a} - \frac{2x a^x}{\log^2 a} + \frac{2a^x}{\log^3 a}$$

$$80. \int x e^{ax} dx = \frac{x e^{ax}}{a} - \frac{e^{ax}}{a^2}$$

$$81. \int x^2 e^{ax} dx = \frac{x^2 e^{ax}}{a} - \frac{2x e^{ax}}{a^2} + \frac{2e^{ax}}{a^3}$$

$$82. \int \sqrt{x} e^{-2\sqrt{x}} dx = -x e^{-2\sqrt{x}} - \sqrt{x} e^{-2\sqrt{x}} - \frac{e^{-2\sqrt{x}}}{2}$$

$$83. \int x^3 e^{\frac{x^2}{2}} dx = x^2 e^{\frac{x^2}{2}} - 2e^{\frac{x^2}{2}}$$

$$84. \int e^{ax} \cos(bx) dx = \frac{e^{ax} [a \cos(bx) + b \sin(bx)]}{a^2 + b^2}$$

$$85. \int e^{ax} \sin(bx) dx = \frac{e^{ax} [a \sin(bx) - b \cos(bx)]}{a^2 + b^2}$$

$$86. \int x^n e^{\lambda x} dx = \frac{x^n e^{\lambda x}}{\lambda} - \frac{n}{\lambda} \int x^{n-1} e^{\lambda x} dx$$

$$87. \int x^n e^x dx = e^x [x^n - nx^{n-1} + n(n-1)x^{n-2} - \dots + (-1)^n n!]$$

$$88. \int e^{\arcsin x} dx = \frac{1}{2} e^{\arcsin x} (x + \sqrt{1-x^2})$$

$$89. \int \operatorname{sen} x \cos x e^{\operatorname{sen} x} dx = \int \frac{\operatorname{sen}(2x)}{2} e^{\operatorname{sen} x} dx = \operatorname{sen} x e^{\operatorname{sen} x} - e^{\operatorname{sen} x}$$

$$90. \int x e^{-x^2} dx = -\frac{e^{-x^2}}{2}$$

$$91. \int \frac{1}{1+e^x} dx = x - \log(1+e^x)$$

$$92. \int \frac{e^x - 1}{e^x + 1} dx = \log(e^x + e^{-x} + 2)$$

$$93. \int \frac{e^{2x}}{\sqrt{1+e^x}} dx = \frac{2}{3} (e^x - 2) \sqrt{1+e^x}$$

$$94. \int \frac{\operatorname{sen} x}{e^x} dx = -\frac{\operatorname{sen} x + \cos x}{2e^x}$$

$$95. \int \frac{1}{1+e^{2x}} dx = \frac{1}{2} \log\left(\frac{e^{2x}}{e^{2x} + 1}\right)$$

$$96. \int \sqrt{e^x - 1} dx = 2\sqrt{e^x - 1} - 2 \operatorname{arctg} \sqrt{e^x - 1}$$

$$97. \int \operatorname{cosh}(ax) dx = \frac{\operatorname{senh}(ax)}{a}$$

$$98. \int x \operatorname{cosh}(ax) dx = \frac{x}{a} \operatorname{senh}(ax) - \frac{\operatorname{cosh}(ax)}{a^2}$$

$$99. \int \frac{1}{\operatorname{cosh}(ax)} dx = \frac{2}{a} \operatorname{arctg}(e^{ax})$$

$$100. \int \frac{1}{\operatorname{cosh}^2 x} dx = \operatorname{tgh} x$$

$$101. \int \operatorname{senh}(ax) dx = \frac{\operatorname{cosh}(ax)}{a}$$

$$102. \int x \operatorname{senh}(ax) dx = \frac{x}{a} \operatorname{cosh}(ax) - \frac{\operatorname{senh}(ax)}{a^2}$$

$$103. \int \frac{1}{\operatorname{senh}(ax)} dx = \frac{1}{a} \log \left| \operatorname{tgh} \left(\frac{ax}{2} \right) \right|$$

$$104. \int \frac{1}{\operatorname{senh}^2 x} dx = -\operatorname{cotgh} x$$

$$105. \int \operatorname{sen}(a x) \cdot \cos(a x) dx = \frac{\operatorname{sen}^2(a x)}{2 a}$$

$$106. \int \operatorname{sen}(a x) \cdot \cos(b x) dx = -\frac{\cos(a+b)x}{2(a+b)} - \frac{\cos(a-b)x}{2(a-b)}$$

$$107. \int \operatorname{sen}(a x) \cdot \operatorname{sen}(b x) dx = \frac{\operatorname{sen}(a-b)x}{2(a-b)} - \frac{\operatorname{sen}(a+b)x}{2(a+b)}$$

$$108. \int \cos(a x) \cdot \cos(b x) dx = \frac{\operatorname{sen}(a-b)x}{2(a-b)} + \frac{\operatorname{sen}(a+b)x}{2(a+b)}$$

$$109. \int \operatorname{sen} x \cos x dx = \frac{\operatorname{sen}^2 x}{2}$$

$$110. \int \operatorname{sen}^2 x \cos x dx = \frac{\operatorname{sen}^3 x}{3}$$

$$111. \int \cos^2 x \operatorname{sen} x dx = -\frac{\cos^3 x}{3}$$

$$112. \int \operatorname{sen}^n x \cos x dx = \frac{\operatorname{sen}^{n+1} x}{n+1}$$

$$113. \int \cos^n x \operatorname{sen} x dx = -\frac{\cos^{n+1} x}{n+1}$$

$$114. \int \frac{1}{\operatorname{sen} x \cos x} dx = \log | \operatorname{tg} x |$$

$$115. \int \frac{1}{\operatorname{sen}^2 x \cos^2 x} dx = \operatorname{tg} x - \operatorname{cotg} x$$

$$116. \int \frac{\operatorname{sen}^2 x - \cos^2 x}{\operatorname{sen}^2 x \cos^2 x} dx = \operatorname{tg} x + \operatorname{cotg} x$$

$$117. \int \frac{\operatorname{sen} x}{\cos^2 x} dx = \frac{1}{\cos x}$$

$$118. \int \frac{\cos x}{\operatorname{sen}^2 x} dx = -\frac{1}{\operatorname{sen} x}$$

$$119. \int \operatorname{sen}(\log x) dx = \frac{x}{2} [\operatorname{sen}(\log x) - \cos(\log x)]$$

$$120. \int \frac{\operatorname{sen} x \cos x}{\sqrt{1 + \cos^2 x}} dx = -\sqrt{1 + \cos^2 x}$$

$$121. \int \frac{\operatorname{sen} x}{\cos^3 x} dx = \frac{1}{2\cos^2 x}$$

$$122. \int \frac{\operatorname{sen}^4 x}{\cos x} dx = -\frac{\operatorname{sen}^3 x}{3} - \operatorname{sen} x - \frac{1}{2} \log \left| \frac{\operatorname{sen} x - 1}{\operatorname{sen} x + 1} \right|$$

$$123. \int \frac{\operatorname{sen}^2 x}{\operatorname{sen} x \cos x + 1} dx = -\frac{1}{2} \log(1 + \operatorname{sen} x \cos x) + \frac{1}{\sqrt{3}} \operatorname{arctg} \left(\frac{2 \operatorname{tg} x + 1}{\sqrt{3}} \right)$$

$$124. \int \sqrt{\frac{\cos x}{\operatorname{sen} x}} dx = \int \sqrt{\cot x} dx = \int \frac{1}{\sqrt{\operatorname{tg} x}} dx = \\ = \frac{1}{\sqrt{2}} \log(\operatorname{sen} x + \cos x + \sqrt{\operatorname{sen} 2x}) + \frac{1}{\sqrt{2}} \operatorname{arcsen}(\operatorname{sen} x - \cos x)$$

$$125. \int \frac{\operatorname{sen} x}{\sqrt{\cos x}} dx = -2\sqrt{\cos x}$$

$$126. \int \frac{x^2}{(x \cos x - \operatorname{sen} x)^2} dx = \frac{x \operatorname{sen} x + \cos x}{x \cos x - \operatorname{sen} x}$$

$$127. \int \frac{\operatorname{sen} x - \operatorname{sen}^3 x}{1 + \operatorname{sen} x} dx = \frac{1}{2} \operatorname{sen} x \cos x - \cos x - \frac{x}{2}$$

$$128. \int \frac{\operatorname{sen}^2 x}{1 - \cos x} dx = x + \operatorname{sen} x$$

$$129. \int \frac{\cos 2x}{\operatorname{sen} x + \cos x} dx = \operatorname{sen} x + \cos x$$

$$130. \int \frac{1 + \operatorname{sen} 2x}{\cos^2 x} dx = \operatorname{tg} x - 2 \log |\cos x|$$

$$131. \int \frac{1 + \operatorname{sen} x}{1 + \cos x} dx = \operatorname{tg} \frac{x}{2} + \log \left(1 + \operatorname{tg}^2 \left(\frac{x}{2} \right) \right)$$

$$132. \int \frac{1 + \operatorname{sen} x}{1 + \cos x} e^x dx = e^x \operatorname{tg} \frac{x}{2}$$

$$133. \int \frac{1 - \cos x}{1 + \cos x} dx = 2 \operatorname{tg} \frac{x}{2} - x$$

$$134. \int \frac{1 - \operatorname{sen} x}{x + \cos x} dx = \log(x + \cos x)$$

$$135. \int \frac{x + \cos x}{1 - \operatorname{sen} x} dx = x \left(\frac{1 + \operatorname{sen} x}{\cos x} \right)$$

$$136. \int \frac{\cos x}{1 + \operatorname{sen}^2 x} dx = \operatorname{arctg}(\operatorname{sen} x)$$

$$137. \int \frac{1}{\operatorname{sen}(ax) \pm \cos(ax)} dx = \frac{1}{a\sqrt{2}} \log \left| \operatorname{tg} \left(\frac{ax}{2} \pm \frac{\pi}{8} \right) \right|$$

$$138. \int \frac{\cos(ax)}{\operatorname{sen}(ax) \pm \cos(ax)} dx = \pm \frac{x}{2} + \frac{1}{2a} \log |\operatorname{sen}(ax) \pm \cos(ax)|$$

$$139. \int \frac{\operatorname{sen}(ax)}{\operatorname{sen}(ax) \pm \cos(ax)} dx = \frac{x}{2} \mp \frac{1}{2a} \log |\operatorname{sen}(ax) \pm \cos(ax)|$$

$$140. \int \frac{1}{a + b \cos x} dx = \begin{cases} 1) \text{ se } a > |b| & \frac{1}{\sqrt{a^2 - b^2}} \operatorname{arctg} \left(\sqrt{\frac{a+b}{a-b}} \cdot \operatorname{tg} \frac{x}{2} \right) \\ 2) \text{ se } b > |a| & \frac{1}{\sqrt{b^2 - a^2}} \log \left(\frac{\operatorname{tg} \frac{x}{2} + \sqrt{\frac{b+a}{b-a}}}{\operatorname{tg} \frac{x}{2} \cdot \sqrt{\frac{b+a}{b-a}}} \right) \end{cases}$$

$$141. \int \frac{1}{a \cos x + b \operatorname{sen} x} dx = \frac{1}{\sqrt{a^2 + b^2}} \log \left(\frac{a \operatorname{tg} \frac{x}{2} - b + \sqrt{a^2 + b^2}}{a \operatorname{tg} \frac{x}{2} - b - \sqrt{a^2 + b^2}} \right)$$

$$142. \int \operatorname{tg}(ax) dx = -\frac{1}{a} \log |\cos(ax)|$$

$$143. \int \operatorname{tg}^2 x dx = \operatorname{tg} x - x$$

$$144. \int \operatorname{tg}^4 x dx = \frac{1}{3} \operatorname{tg}^3 x - \operatorname{tg} x + x$$

$$145. \int \operatorname{tg}^n x \, dx = \int \frac{1}{\operatorname{cotg}^n x} \, dx = \frac{\operatorname{tg}^{(n-1)} x}{(n-1)} - \int \operatorname{tg}^{n-2} x \, dx$$

$$146. \int \frac{\operatorname{tg}^n x}{1 + \operatorname{sen}^2 x} \, dx = \frac{\operatorname{tg}^{(n+1)}(a x)}{a(n+1)}$$

$$147. \int \frac{\operatorname{tg} x}{1 + \operatorname{sen}^2 x} \, dx = \frac{1}{4} \log(2 \operatorname{tg}^2 x + 1)$$

$$148. \int \frac{\operatorname{tg}^3 x + \operatorname{tg} x}{\operatorname{tg} x + 4} \, dx = \operatorname{tg} x - 4 \log|4 + \operatorname{tg} x|$$

$$149. \int \operatorname{cotg}(a x) \, dx = \frac{1}{a} \log|\operatorname{sen}(a x)|$$

$$150. \int \operatorname{cotg}^2 x \, dx = -\operatorname{cotg} x - x$$

$$151. \int \operatorname{cotg}^n x \, dx = \int \frac{1}{\operatorname{tg}^n x} \, dx = -\frac{\operatorname{cotg}^{n-1} x}{(n-1)} - \int \operatorname{cotg}^{(n-2)} x \, dx$$

$$152. \int \operatorname{arccosh}\left(\frac{x}{a}\right) \, dx = \begin{cases} 1) \text{ per } \operatorname{arccosh}\left(\frac{x}{a}\right) > 0 & x \operatorname{arccosh}\left(\frac{x}{a}\right) - \sqrt{x^2 - a^2} \\ 2) \text{ per } \operatorname{arccosh}\left(\frac{x}{a}\right) < 0 & x \operatorname{arccosh}\left(\frac{x}{a}\right) + \sqrt{x^2 - a^2} \end{cases}$$

$$153. \int \operatorname{arcsenh}\left(\frac{x}{a}\right) \, dx = x \operatorname{arcsenh}\left(\frac{x}{a}\right) - \sqrt{x^2 + a^2}$$

$$154. \int \operatorname{tgh}(a x) \, dx = \frac{1}{a} \log|\cosh(a x)|$$

$$155. \int \operatorname{arctgh}\left(\frac{x}{a}\right) \, dx = x \operatorname{arctgh}\left(\frac{x}{a}\right) + \frac{a}{2} \log|a^2 - x^2|$$

$$156. \int \operatorname{arccotgh}\left(\frac{x}{a}\right) \, dx = x \operatorname{arccotgh}\left(\frac{x}{a}\right) + \frac{a}{2} \log|x^2 - a^2|$$

$$157. \int \operatorname{cotgh}(a x) \, dx = \frac{1}{a} \log|\operatorname{senh}(a x)|$$

$$158. \int \cos(a x) \, dx = \frac{1}{a} \operatorname{sen}(a x)$$

$$159. \int x \cos(ax) dx = \frac{1}{a^2} \cos(ax) + \frac{x}{a} \sin(ax)$$

$$160. \int x^2 \cos x dx = x^2 \sin x + 2x \cos x - 2 \sin x$$

$$161. \int \cos(\sqrt{x}) dx = 2\sqrt{x} \sin \sqrt{x} + 2 \cos \sqrt{x}$$

$$162. \int \cos^2 x dx = \frac{x}{2} + \frac{1}{4} \sin(2x) = \frac{1}{2} \sin x \cos x + \frac{x}{2}$$

$$163. \int \cos^3 x dx = \frac{1}{3} \cos^2 x \sin x + \frac{2}{3} \sin x = \sin x - \frac{1}{3} \sin^3 x$$

$$164. \int \frac{1}{\cos(ax)} dx = \frac{1}{a} \log \left| \operatorname{tg} \left(\frac{ax}{2} + \frac{\pi}{4} \right) \right|$$

$$165. \int \frac{1}{1 + \cos(ax)} dx = \frac{1}{a} \operatorname{tg} \left(\frac{ax}{2} \right)$$

$$166. \int \frac{1}{1 - \cos(ax)} dx = -\frac{1}{a} \operatorname{cotg} \left(\frac{ax}{2} \right)$$

$$167. \int \frac{1}{\cos^2 x} dx = \operatorname{tg} x$$

$$168. \int \frac{1}{\cos^4 x} dx = \operatorname{tg} x + \frac{1}{3} \operatorname{tg}^3 x$$

$$169. \int \frac{1}{\cos^n x} dx = \frac{\sin x}{(n-1)\cos^{(n-1)}x} + \frac{n-2}{n-1} \int \frac{1}{\cos^{n-2} x} dx$$

$$170. \int \cos^n x dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x dx$$

$$171. \int \frac{x}{1 + \cos(ax)} dx = \frac{x}{a} \operatorname{tg} \left(\frac{ax}{2} \right) + \frac{2}{a^2} \log \left| \cos \left(\frac{ax}{2} \right) \right|$$

$$172. \int \frac{x}{1 - \cos(ax)} dx = -\frac{x}{a} \operatorname{cotg} \left(\frac{ax}{2} \right) + \frac{2}{a^2} \log \left| \sin \left(\frac{ax}{2} \right) \right|$$

$$173. \int \frac{\cos(ax)}{1 + \cos(ax)} dx = x - \frac{1}{a} \operatorname{tg} \left(\frac{ax}{2} \right)$$

$$174. \int \frac{\cos(ax)}{1 - \cos(ax)} dx = -x - \frac{1}{a} \cotg\left(\frac{ax}{2}\right)$$

$$175. \int \frac{x}{\cos^2 x} dx = x \operatorname{tg} x + \log |\cos x|$$

$$176. \int \frac{x}{\cos^2 x^2} dx = \frac{1}{2} \operatorname{tg}(x^2)$$

$$177. \int x^m \cos x dx = x^m \operatorname{sen} x - m \int x^{m-1} \operatorname{sen} x dx$$

$$178. \int \cos(\log x) dx = \frac{x}{2} [\operatorname{sen}(\log x) + \cos(\log x)]$$

$$179. \int \operatorname{sen}(ax) dx = -\frac{1}{a} \cos(ax)$$

$$180. \int x \operatorname{sen}(ax) dx = \frac{1}{a^2} \operatorname{sen}(ax) - \frac{x}{a} \cos(ax)$$

$$181. \int x^2 \operatorname{sen} x dx = -x^2 \cos x + 2x \operatorname{sen} x + 2 \cos x$$

$$182. \int \operatorname{sen}^2 x dx = \frac{x}{2} - \frac{1}{4} \operatorname{sen} 2x = \frac{x}{2} - \frac{1}{2} \operatorname{sen} x \cos x$$

$$183. \int \operatorname{sen}^3 x dx = -\cos x + \frac{1}{3} \cos^3 x$$

$$184. \int \frac{1}{\operatorname{sen}(ax)} dx = \frac{1}{a} \log \left| \operatorname{tg} \left(\frac{ax}{2} \right) \right|$$

$$185. \int \frac{1}{1 + \operatorname{sen}(ax)} dx = \frac{1}{a} \operatorname{tg} \left(\frac{ax}{2} - \frac{\pi}{4} \right)$$

$$186. \int \frac{1}{1 + \operatorname{sen} x} dx = -\frac{2}{1 + \operatorname{tg} \frac{x}{2}}$$

$$187. \int \frac{1}{1 - \operatorname{sen}(ax)} dx = \frac{1}{a} \operatorname{tg} \left(\frac{ax}{2} + \frac{\pi}{4} \right)$$

$$188. \int \frac{1}{1 - \operatorname{sen} x} dx = \frac{2}{1 - \operatorname{tg} \frac{x}{2}}$$

$$189. \int \frac{x}{1 + \operatorname{sen}(ax)} dx = \frac{x}{a} \operatorname{tg} \left(\frac{ax}{2} - \frac{\pi}{4} \right) + \frac{2}{a^2} \log \left[\cos \left(\frac{ax}{2} - \frac{\pi}{4} \right) \right]$$

$$190. \int \frac{x}{1 - \operatorname{sen}(ax)} dx = \frac{x}{a} \operatorname{cotg} \left(\frac{\pi}{4} - \frac{ax}{2} \right) + \frac{2}{a^2} \log \left| \operatorname{sen} \left(\frac{\pi}{4} - \frac{ax}{2} \right) \right|$$

$$191. \int \frac{\operatorname{sen}(ax)}{1 + \operatorname{sen}(ax)} dx = x + \frac{1}{a} \operatorname{tg} \left(\frac{\pi}{4} - \frac{ax}{2} \right)$$

$$192. \int \frac{\operatorname{sen}(ax)}{1 - \operatorname{sen}(ax)} dx = -x + \frac{1}{a} \operatorname{tg} \left(\frac{\pi}{4} + \frac{ax}{2} \right)$$

$$193. \int \frac{1}{\operatorname{sen}^n x} dx = -\frac{\cos x}{(n-1)\operatorname{sen}^{n-1} x} + \frac{n-2}{n-1} \int \frac{1}{\operatorname{sen}^{n-2} x} dx$$

$$194. \int \frac{1}{\operatorname{sen}^2 x} dx = -\operatorname{cotg} x$$

$$195. \int \operatorname{sen}^n x dx = -\frac{1}{n} \operatorname{sen}^{n-1} x \cos x + \frac{n-1}{n} \int \operatorname{sen}^{n-2} x dx$$

$$196. \int \operatorname{sen}^5 x dx = \frac{1}{5} \cos^5 x + \frac{2}{3} \cos^3 x - \cos x$$

$$197. \int x^n \operatorname{sen} x dx = -x^n \cos x + n \int x^{n-1} \cos x dx$$

$$198. \int \operatorname{sen}^{(2n+1)} x dx = \int (\operatorname{sen}^2 x)^n \operatorname{sen} x dx = - \int (1 - \cos^2 x)^n d(\cos x) \text{ si sviluppa il binomio e si}$$

arriva ad integrali del tipo $\int \cos^{2n} x d(\cos x) = \frac{\cos^{(2n+1)} x}{2n+1}$

$$199. \int \operatorname{sen}^{(2m+1)} x \cos^n x dx = \int (\operatorname{sen}^2 x)^m \operatorname{sen} x \cos^n x dx = \int (1 - \cos^2 x)^m \cos^n x d(\cos x) \text{ si}$$

riduce ad integrali del tipo $\int \cos^{(2m+n)} x d(\cos x) = \frac{\cos^{(2m+n+1)} x}{2m+n+1}$

$$200. \int \operatorname{sen}^{(2m)} x \cos^{(2n)} x dx = \int \operatorname{sen}^{(2m)} x (1 - \operatorname{sen}^2 x)^n dx \text{ si riduce ad integrali del}$$

tipo $\int \operatorname{sen}^{2p} x dx$

$$201. \int \sin^n x \cos^n x \, dx = \int \left[\frac{1}{2} \sin(2x) \right]^n dx \stackrel{\text{(ponendo } 2x=t)}{=} \frac{1}{2^{n+1}} \int \sin^n t \, dt$$

$$202. \int \arcsen\left(\frac{x}{a}\right) dx = x \arcsen\left(\frac{x}{a}\right) + \sqrt{a^2 - x^2}$$

$$203. \int \arcsen^2 x \, dx = x \arcsen^2 x + 2\sqrt{1-x^2} \arcsen x - 2x$$

$$204. \int \arcsen^3 x \, dx = x \arcsen^3 x + 3\sqrt{1-x^2} \arcsen^2 x - (6 \arcsen x)x + 6\sqrt{1-x^2}$$

$$205. \int \arcsen \sqrt{x} \, dx = \left(x - \frac{1}{2}\right) \arcsen \sqrt{x} + \frac{1}{2} \sqrt{x-x^2}$$

$$206. \int x \arcsen\left(\frac{x}{a}\right) dx = \left(\frac{x^2}{2} - \frac{a^2}{4}\right) \arcsen\left(\frac{x}{a}\right) + \frac{x}{4} \sqrt{a^2 - x^2}$$

$$207. \int x \arcsen(x^2) dx = \frac{x^2}{2} \arcsen(x^2) + \frac{1}{2} \sqrt{1-x^4}$$

$$208. \int \arcsen^n x \, dx = x \arcsen^n x - n \int \frac{x \arcsen^{n-1} x}{\sqrt{1-x^2}} dx$$

$$209. \int \arccos\left(\frac{x}{a}\right) dx = x \arccos\left(\frac{x}{a}\right) - \sqrt{a^2 - x^2}$$

$$210. \int x \arccos\left(\frac{x}{a}\right) dx = \left(\frac{x^2}{2} - \frac{a^2}{4}\right) \arccos\left(\frac{x}{a}\right) - \frac{x}{4} \sqrt{a^2 - x^2}$$

$$211. \int \operatorname{arccotg}\left(\frac{x}{a}\right) dx = x \operatorname{arccotg}\left(\frac{x}{a}\right) + \frac{a}{2} \log(a^2 + x^2)$$

$$212. \int x \operatorname{arccotg}\left(\frac{x}{a}\right) dx = \frac{1}{2} (x^2 + a^2) \operatorname{arccotg}\left(\frac{x}{a}\right) + \frac{ax}{2}$$

$$213. \int \operatorname{arctg}\left(\frac{x}{a}\right) dx = x \operatorname{arctg}\left(\frac{x}{a}\right) - \frac{a}{2} \log(a^2 + x^2)$$

$$214. \int x \operatorname{arctg}\left(\frac{x}{a}\right) dx = \frac{1}{2} (x^2 + a^2) \operatorname{arctg}\left(\frac{x}{a}\right) - \frac{ax}{2}$$

$$215. \int x \operatorname{arctg}^2 x \, dx = \frac{x^2+1}{2} \operatorname{arctg}^2 x - x \operatorname{arctg} x + \frac{1}{2} \log(1+x^2)$$

$$216. \int \operatorname{arctg} \sqrt{x} \, dx = (x+1) \operatorname{arctg} \sqrt{x} - \sqrt{x}$$

$$217. \int \operatorname{arctg} \left(\frac{x-1}{x+1} \right) dx = x \operatorname{arctg} \left(\frac{x-1}{x+1} \right) - \log \sqrt{1+x^2}$$

$$218. \int \frac{1}{(1+x^2) \operatorname{arctg} x} dx = \log |\operatorname{arctg} x|$$

$$219. \int \frac{\cotg^n(ax)}{\operatorname{sen}^2(ax)} dx = -\frac{\cotg^{n+1}(ax)}{a(n+1)}$$

$$220. \int \frac{x}{1+x^4} dx = \frac{1}{2} \operatorname{arctg} x^2$$

$$221. \int \frac{1}{1-x^3} dx = \frac{1}{3} \log \left(\frac{\sqrt{x^2+x+1}}{1-x} \right) + \frac{1}{\sqrt{3}} \operatorname{arctg} \left(\frac{2x+1}{\sqrt{3}} \right)$$

$$222. \int \frac{1}{x} dx = \log x$$

$$223. \int \log x \, dx = x \log x - x$$

$$224. \int \log^2 x \, dx = x (\log^2 x - 2 \log x + 2)$$

$$225. \int \log^n x \, dx = x \log^n x - n \int \log^{(n-1)} x \, dx$$

$$226. \int \log(a+bx) \, dx = \frac{a+bx}{b} \log(a+bx) - \frac{(a+bx)}{b}$$

$$227. \int \log(x^2) \, dx = x \log(x^2) - 2x$$

$$228. \int x \log x \, dx = \frac{x^2}{2} \left(\log x - \frac{1}{2} \right)$$

$$229. \int x^2 \log x \, dx = \frac{1}{3} \left(x^3 \log x - \frac{x^3}{3} \right)$$

$$230. \int x^3 \log x \, dx = \frac{x^4}{4} \log x - \frac{x^4}{16}$$

$$231. \int x^n \log x \, dx = \frac{x^{n+1}}{n+1} \left(\log x - \frac{1}{n+1} \right)$$

$$232. \int \frac{\log x}{x} \, dx = \frac{1}{2} \log^2 x$$

$$233. \int \frac{\log x}{x^2} \, dx = -\frac{\log x}{x} - \frac{1}{x}$$

$$234. \int \frac{\log^3 x}{x} \, dx = \frac{1}{4} \log^4 x$$

$$235. \int \frac{\log^n x}{x} \, dx = \frac{\log^{(n+1)} x}{n+1}$$

$$236. \int \frac{1}{x \log x} \, dx = \log |\log x|$$

$$237. \int \frac{1}{x \sqrt{\log x}} \, dx = 2 \sqrt{\log x}$$

$$238. \int \frac{\log(\log x)}{x} \, dx = \log x [\log(\log x) - 1]$$

$$239. \int \frac{1}{x \sqrt{1 - \log^2 x}} \, dx = \arcsen(\log x)$$

$$240. \int \log(\sqrt{1+x} + \sqrt{1-x}) \, dx = x \log(\sqrt{1+x} + \sqrt{1-x}) + \frac{1}{2} \arcsen x - \frac{x}{2}$$

$$241. \int \operatorname{sen}(\log x) \, dx = \frac{x}{2} [\operatorname{sen}(\log x) - \cos(\log x)]$$

$$242. \int \cos(\log x) \, dx = \frac{x}{2} [\operatorname{sen}(\log x) + \cos(\log x)]$$

$$243. \int \frac{\cos(\log x)}{x} \, dx = \operatorname{sen}(\log x)$$

$$244. \int \log(x^2 + a^2) \, dx = x \log(x^2 + a^2) - 2x + 2a \operatorname{arctg} \frac{x}{a}$$

$$245. \int \log(x^2 - a^2) \, dx = x \log(x^2 - a^2) - 2x + a \log \left(\frac{x+a}{x-a} \right)$$

$$246. \int \log(x + \sqrt{1+x^2}) dx = x \log(x + \sqrt{1+x^2}) - \sqrt{1+x^2}$$

$$247. \int \sqrt[3]{x^2} \log x dx = \frac{3}{5} \sqrt[3]{x^5} \left(\log x - \frac{3}{5} \right)$$

$$248. \int e^x \log(1+e^x) dx = e^x \log(1+e^x) + \log(1+e^x) - e^x$$

$$249. \int \log_a x dx = x(\log_a x - \log_a e)$$

$$250. \int \frac{1}{(1+x^2)^n} dx = \frac{x}{(2n-2)(1+x^2)^{n-1}} + \frac{2n-3}{2n-2} \int \frac{1}{(1+x^2)^{n-1}} dx$$

$$251. \int \frac{1}{x^2 + px + q} dx = \begin{cases} \text{Caso } \Delta = 0 & = \frac{-1}{x + \frac{p}{2}} \\ \text{Caso } \Delta < 0 & = \frac{2}{\sqrt{-\Delta}} \operatorname{arctg} \frac{2x+p}{\sqrt{-\Delta}} \\ \text{Caso } \Delta > 0 & \text{si trovano le radici e si ottiene :} \\ & - \frac{2}{\sqrt{p^2 - 4q}} \operatorname{settg} \left(\frac{2x+p}{\sqrt{p^2 - 4q}} \right) + c \end{cases}$$

$$252. \int \frac{1}{\sqrt{x^2 + px + q}} dx = \begin{cases} \text{Caso } \Delta < 0 & = \operatorname{settsenh} \left(\frac{2x+p}{\sqrt{4q-p^2}} \right) + c \\ \text{Caso } \Delta > 0 & = \operatorname{settcosh} \left(\frac{2x+p}{\sqrt{p^2-4q}} \right) + c \end{cases}$$

$$253. \int \frac{1}{\sqrt{-x^2 + px + q}} dx = \boxed{\text{Caso } \Delta > 0} = \arcsen\left(\frac{2x - p}{\sqrt{4q + p^2}}\right) + c$$

$$254. \int \frac{ax + b}{x^2 + px + q} dx = \begin{cases} \boxed{\text{Caso } \Delta > 0} & = \text{ se } x_1 \text{ e } x_2 \text{ sono le radici} \\ & \text{si ha } = \frac{ax_1 + b}{x_1 - x_2} \log|x - x_1| + \frac{ax_2 + b}{x_2 - x_1} \log|x - x_2| \\ \boxed{\text{Caso } \Delta < 0} & = \frac{a}{2} \log(x^2 + px + q) + \frac{2b - ap}{\sqrt{-\Delta}} \operatorname{arctg}\left(\frac{2x + p}{\sqrt{-\Delta}}\right) \end{cases}$$

$$255. \int \frac{x}{(ax^2 + bx + c)} dx = \frac{1}{2a} \log(ax^2 + bx + c) - \frac{b}{2a} \int \frac{1}{ax^2 + bx + c} dx$$

$$256. \int \frac{1}{(x^2 + px + q)^n} dx = \left(\frac{2}{\sqrt{-\Delta}}\right)^{2n-1} \cdot \int \frac{d\left(\frac{2x+p}{\sqrt{-\Delta}}\right)}{\left[1 + \left(\frac{2x+p}{\sqrt{-\Delta}}\right)^2\right]^n} \text{ che si riconduce ad integrali del}$$

tipo: $a \int \frac{1}{(1+t^2)^n} dt$

$$257. \int \frac{ax + b}{(x^2 + px + q)^n} dx = \frac{a}{2} \int \frac{2x + p}{(x^2 + px + q)^n} dx + \left(b - \frac{ap}{2}\right) \int \frac{1}{(x^2 + px + q)^n} dx$$

$$258. \int \frac{1}{\sqrt{ax^2 + bx + c}} dx = \begin{cases} \text{caso } a > 0 & = \frac{1}{\sqrt{a}} \log \left| x + \frac{b}{2a} + \sqrt{x^2 + \frac{b}{a}x + \frac{c}{a}} \right| \\ \text{caso } a < 0, \Delta > 0 & = \frac{1}{\sqrt{|a|}} \arcsen \frac{2|a|x - b}{\sqrt{\Delta}} = \text{ se } x_1 \text{ e } x_2 \\ & \text{sono le radici } = -\frac{2}{\sqrt{|a|}} \operatorname{arctg} \sqrt{\frac{x_2 - x}{x - x_1}} \end{cases}$$

$$259. \int_0^x e^{-x^2} dx = x - \frac{x^3}{3 \cdot 1!} + \frac{x^5}{2! \cdot 5} + \dots + (-1)^{n-1} \frac{x^{2n-1}}{(n-1)!(2n-1)}$$

$$260. \int_0^{+\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$$

$$261. \int_0^x \frac{\text{sen} x}{x} dx = x - \frac{x^3}{3! \cdot 3} + \frac{x^5}{5! \cdot 5} + \dots + (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!(2n-1)}$$

$$262. \int_0^1 x^x dx = 1 - \frac{1}{2^2} + \frac{1}{3^3} - \dots + (-1)^{n-1} \frac{1}{n^n}$$

$$263. \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{1-k^2 \text{sen}^2 x}} dx = \frac{\pi}{2} \left[1 + \left(\frac{1}{2}\right)^2 k^2 + \left(\frac{1 \times 3}{2 \times 4}\right)^2 k^4 + \left(\frac{1 \times 3 \times 5}{2 \times 4 \times 6}\right)^2 k^6 + \dots \right]$$

dove $-1 < k < 1$

$$264. \int_0^{\frac{\pi}{2}} \sqrt{1-k^2 \text{sen}^2 x} dx = \frac{\pi}{2} \left[1 - \left(\frac{1}{2}\right)^2 k^2 - \frac{1}{3} \left(\frac{1 \times 3}{2 \times 4}\right)^2 k^4 - \frac{1}{5} \left(\frac{1 \times 3 \times 5}{2 \times 4 \times 6}\right)^2 k^6 + \dots \right]$$

$$265. \int_0^{\frac{\pi}{2}} \text{sen}^{2n+1} x dx = \int_0^{\frac{\pi}{2}} \cos^{2n+1} x dx = \frac{2 \times 4 \times 6 \times 8 \times \dots \times (2n)}{3 \times 5 \times 7 \times 9 \times \dots \times (2n+1)}$$

$$266. \int_0^{\frac{\pi}{2}} \text{sen}^{2n} x dx = \int_0^{\frac{\pi}{2}} \cos^{2n} x dx = \frac{1 \times 3 \times 5 \times \dots \times (2n-1)}{2 \times 4 \times 6 \times \dots \times (2n)} \times \frac{\pi}{2}$$